APPENDIX D-2. WATER HEATER ANALYSIS MODEL (WHAM)

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APPENDIX D-2. WATER HEATER ANALYSIS MODEL (WHAM)

D-2.1 INTRODUCTION

The Water Heater Analysis Model (WHAM) is a simple energy equation that accounts for a variety of operating conditions and water heater characteristics when calculating energy consumption.[1] The WHAM energy calculation is based on assumptions that account for a variety of field conditions and water heater types. The results of WHAM calculations have been compared to the results of detailed water heaters simulation programs and show a high level of accuracy in estimating energy consumption.

D-2.2 ALGORITHM FUNDAMENTALS

WHAM, based on the DOE water heater test procedure, permits the user to minimally describe both the water heater and the water heater operating conditions.[2] The operating conditions are characterized by four variables:

- daily draw volume,
- thermostat setpoint temperature,
- inlet water temperature, and
- ambient air temperature.

The water heater is described by:

- rated input (Pon),
- stand-by heat loss coefficient (UA), and
- recovery efficiency (RE).

The DOE water heater test procedure includes a method for calculating the Energy Factor (EF), a measure of efficiency. During the standard test procedure, a 24-hour trial is performed in which 6 equal draws are made during the first 6 hours of the test for a total of 64.3 gallons (243.4L). For the remaining 18 hours, the water heater is in stand-by mode and associated energy losses are measured. The energy consumption during the test is totalled and the mass of water drawn is summed over the six draws. The temperature of the water entering, leaving, and inside the water heater tank is recorded. The temperature of the air around the water heater is also noted. Additional equations appear in the test procedure to correct for any deviation from the standard testing specifications to guarantee uniformity in testing results in different laboratories. The test is based on the conditions shown in Table D-2.1.

Table D-2.1 DOE Test Procedure Operating Conditions

Daily Draw *gal (liter)*	Thermostat Setpoint Temperature *EF (EC)*	Inlet Water Temperature *EF (EC)*	Ambient Air Temperature *EF (EC)*
64.3 (243.4)	135 (57)	58 (14)	67.5 (49.7)

Stated in general terms, EF is the ratio of the energy output of the water heater to the total amount of energy delivered to the water heater. More specifically, EF is the added energy content of the water drawn from the water heater divided by the energy required to heat and maintain the water at the water heater's setpoint temperature:

$$EF = \frac{M \cdot C_p \cdot (T_{tank} - T_{inlet})}{Q_{dm}} \qquad \textbf{(Eq. D-2.1)}$$

where:

EF = energy factor
M = mass of water drawn (lbs) (kg)
C_p = specific heat of water, (Btu / lb - EF) (k./kg - EC)
T_{tank} = water heater thermostat setpoint temperature (EF) (EC)
T_{inlet} = the inlet water temperature (EF) (EC)
Q_{dm} = water heater's daily energy consumption (Btu) (kWh)

This equation establishes the EF for water heaters. Manufacturers are required by federal law to determine EF and to label all products with EF. UA and RE are determined in the course of the test procedure and are used in the test procedure equations to calculate EF.

WHAM is based on a number of assumptions. It assumes that the values for Pon, UA, and RE do not change significantly when conditions differ from test procedure conditions. Other assumptions also help to simplify the equation:

- Water temperature in the tank is always at the thermostat setpoint
- Water and air temperatures are constant
- Water density is constant
- Specific heat of water is constant

WHAM only calculates average energy use. Other tools should be used for calculating water heaters' response to peak use and for sizing systems.

D-2.3 DEVELOPMENT

WHAM relies on the following variables, which are defined in the DOE test procedure:

Q_{In} =	total water heater energy consumption (Btu/day) (kWh/day)
Q_{recov} =	energy consumed to heat the water drawn from the water heater (Btu/day) (kWh/day)
Q_{stdby} =	energy consumed to make up for standby heat loss from the water heater (Btu/day) (kWh/day)
Q_{out} =	heat content of the water drawn from the water heater (Btu/day) (kWh/day)
H_{stdby} =	hours of standby for storage water heaters (hr/day)
RE =	recovery efficiency
EF =	energy factor
Pon =	rated input power (Btu/hr) (kWh/day)
UA =	standby heat loss coefficient (Btu/hr - EF) (kWh / EC)
T_{tank} =	tank thermostat setpoint temperature (EF) (EC)
T_{in} =	inlet water temperature (EF) (EC)
T_{amb} =	temperature of ambient air surrounding the water heater (EF) (EC)
vol =	volume of water drawn in 24 hours (gal/day) (liter/day)
den =	density of water (lb/gal) (kg/L)
C_p =	specific heat of water (Btu/lb- EF) (kJ/(kg · EC)

The amount of energy consumed by the water heater (Q_{in}) includes not only the amount of energy needed to heat the water (Q_{recov}), but also the amount of energy to replace that which is lost to the surroundings when no water is being drawn from the tank (Q_{stdby}).

$$Q_{in} = Q_{recov} + Q_{stdby} \quad \textbf{(Eq. D-2.2)}$$

By definition, energy consumed to heat the water drawn from the water heater is equal to the heat content in the water divided by the RE.

$$Q_{recov} = \frac{Q_{out}}{RE} \quad \textbf{(Eq. D-2.3)}$$

Energy consumption to compensate for standby energy loss from a storage tank water heater is equal to the difference in the water heater thermostat setpoint and the average ambient air temperature around the water heater multiplied by the standby heat loss coefficient and the hours of standby. Note that the standby heat loss coefficient is expressed in terms of energy input to the water heater. Standby is the time when the water heater is not heating water to recover from a draw. Time when the burner is firing or elements are energized to make up for standby heat losses is included in the hours of standby.

$$Q_{stdby} = UA \cdot (T_{tank} - T_{amb}) \cdot H_{stdby} \qquad \textbf{(Eq. D-2.4)}$$

The hours of standby variable is the time the water heater is recovering from a draw. This is equal to 24 hours minus the ratio of the amount of energy used to heat the water drawn from the water heater divided by the rated input power of the water heater.

$$H_{stdby} = 24 - \frac{Q_{recov}}{Pon} \qquad \textbf{(Eq. D-2.5)}$$

Heat content of the hot water drawn from the water heater is equal to the volume of water drawn multiplied by the water's density, specific heat, and the temperature difference between the water heater setpoint and the inlet water temperature.

$$Q_{out} = vol \cdot den \cdot C_p \cdot (T_{tank} - T_{in}) \qquad \textbf{(Eq. D-2.6)}$$

The energy factor is equal to the heat content of the water drawn from the heater divided by the total energy consumption.

$$EF = \frac{Q_{out}}{Q_{in}} \qquad \textbf{(Eq. D-2.7)}$$

Q_{in} can be represented by the parameters from DOE test procedure by substituting for Q_{recov} and Q_{stdby} in Eq. D-2.2:

$$Q_{in} = \frac{Q_{out}}{RE} + UA \cdot (T_{tank} - T_{amb}) \cdot \left(24 - \frac{Q_{out}}{RE \cdot Pon} \right) \qquad \textbf{(Eq. D-2.8)}$$

WHAM can now be expressed by expanding Q_{out}, rearranging the terms, and simplifying. This is the equation to calculate average daily energy consumption.

$$Q_{in} = \frac{vol \cdot den \cdot C_p \cdot (T_{tank} - T_{in})}{RE} \cdot \left(1 - \frac{UA \cdot (T_{tank} - T_{amb})}{Pon} \right) + 24 \cdot UA \cdot (T_{tank} - T_{amb}) \qquad \textbf{(Eq. D-2.9)}$$

WHAM is a function of readily available information that can be grouped into three categories:

- Physical characteristics of water represented by density and specific heat (C_p)
- Water heater operating conditions represented by draw volume and temperatures
- Water heater parameters represented by UA, RE, and Pon

D-2.4 CALCULATION OF STANDBY HEAT LOSS COEFFICIENT UA

UA is a necessary input in the WHAM equation, yet it is not always available. The same DOE test procedure that established a basis for the WHAM equation can be used for determining a value for UA.

Since energy consumption to make up for standby losses is equal to the total energy consumption minus the energy content of water withdrawn from the water heater, it is possible to rearrange Eq. D-2.2 as follows:

$$Q_{stdby} = Q_{in} - Q_{recov} \qquad \textbf{(Eq. D-2.10)}$$

Substituting for Q_{in} and Q_{recov} from Eqs. D-2.3 and D-2.6, Eq. D-2.10 becomes:

$$Q_{stdby} = Q_{out} \cdot \left(\frac{1}{EF} - \frac{1}{RE} \right) \qquad \textbf{(Eq. D-2.11)}$$

The energy consumption to make up for standby losses can also be defined by combining Eqs. D-2.3 D-2.4, and D-2.5:

$$Q_{stdby} = UA \cdot (T_{tank} - T_{amb}) \cdot \left(24 - \frac{Q_{out}}{RE \cdot Pon} \right) \qquad \textbf{(Eq. D-2.12)}$$

Equating the two terms for Q_{stdby} from Eq. D-2.11 and D-2.12 and solving for UA produces Eq. D-2.13:

$$UA = \frac{\left(\frac{1}{EF} - \frac{1}{RE}\right)}{(T_{tank} - T_{amb}) \cdot \left(\frac{24}{Q_{out}} - \frac{1}{RE \cdot Pon}\right)} \qquad \textbf{(Eq. D-2.13)}$$

Inserting the temperatures and conditions specified in the test procedure, the equation becomes:

$$UA = \frac{\left(\frac{1}{EF} - \frac{1}{RE}\right)}{67.5 \cdot \left(\frac{24}{41094} - \frac{1}{RE \cdot Pon}\right)} \qquad \textbf{(Eq. D-2.14)}$$

This equation allows the user to determine a value for UA, if it is unavailable from other sources.

D-2.5 SAMPLE CALCULATION

A sample application of the WHAM equation to calculate the energy consumption and the standby heat loss coefficient (UA) is presented in Table D-2.2 and Table D-2.3. The available data is organized into three sections representing: 1) the inputs for the energy parameters from DOE test procedure, 2) the operating conditions parameters, and 3) the physical characteristics of water. The average daily energy input (energy consumption) and the standby heat loss coefficient are calculated directly from the input data. Detailed definitions of all the parameters can be found in the DOE Test Procedure document.[2]

Table D-2.2 WHAM Sample Calculation with Specified Inputs

WHAM Sample Calculation

Inputs:

10 CFR part 430 Testing Procedure Results

UA	3.319 (1.751)	Btu/hr-deg F (W/ deg C)	Standby Heat Loss Coefficient
RE	0.98	[-]	Recovery Efficiency
Pon	15,354 (4500)	Btu/hr (W)	Rated Input Power

Operating Conditions

vol	55 (208)	gal/day (L/day)	Total Volume of Water Removed During the 24 hr Test
Ttank	140 (60)	deg F (deg C)	Thermostat Set Point
Tin	56 (13)	deg F (deg C)	Inlet Water Temperature
Tamb	65 (18)	deg F (deg C)	Ambient Temperature

Physical Characteristiscs of the Water

dens	8.293752 (995.25)	lb/gal (kg/cubic meter)	Density of the Water
M	456.2 (206.9)	lb (kg)	Mass of Water Withdrawn During the 24 hr Test
Cp	1.000743 (4.187)	Btu/lb-deg F (kJ/(kg - deg C)	Specific Heat of Water

Results:

Average Daily Energy Input

Qin	**44468 (13033)**	Btu/day (W/day)	Average Daily Energy Input

where,

$$Qin = \frac{vol \cdot dens \cdot Cp \cdot (Ttank - Tin)}{RE_{DOE}} \cdot \left[1 - \frac{UA_{DOE} \cdot (Ttank - Tamb)}{Pon_{DOE}}\right] + 24 \cdot UA_{DOE} \cdot (Ttank - Tamb)$$

Table D-2.3 UA Sample Calculation with Specified Inputs

Standby Heat Loss Coefficent (UA)
Sample Calculation

Inputs:

10 CFR part 430 Testing Procedure Results

EF	0.700		Energy Factor
RE	0.78		Recovery Efficiency
Pon	40,000 (11.72)	Btu/hr (kW)	Rated Input Power

Operating Conditions

vol	64.3 (243)	gal/day (L/day)	Total Volume of Water Removed During the 24 hr Test
Ttank	135 (57)	deg F (deg C)	Thermostat Set Point
Tin	58 (14.5)	deg F (deg C)	Inlet Water Temperature
Qout	=M*Cp*(Ttank-Tin)	Btu/day (W-h/day)	Energy Content of Water Withdrawn During 24 hr Test

Physical Characteristiscs of the Water at mean of Tin & Tout at test conditions

dens	8.2938 (995.25)	lb/gal (kg/cubic meter)	Density of the Water
M	456.2 (206.9)	lb (kg)	Mass of Water Withdrawn During the 24 hr Test
Cp	1.0007 (4.187)	Btu/lb-deg F (kJ/kg - K)	Specific Heat of Water

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Results:

Standby Heat Loss Coefficient

UA	3.655 (1.928)	Btu/hr-deg F (W/deg C)

where,

$$UA = \frac{\frac{1}{EF} - \frac{1}{RE}}{67.5 \cdot \left(\frac{24}{Qout} - \frac{1}{RE \cdot Pon}\right)} \cdot \frac{BTU}{deg \cdot hr}$$

REFERENCES

1. Lutz, J., C. D. Whitehead, A. Lekov, D. Winiarski, and G. Rosenquist, WHAM: A Simplified Energy Consumption Equation for Water Heaters. In *1998 ACEEE Summer Study on Energy Efficiency in Buildings*. 1998. Asilomar, CA, August 23-28, 1998: American Council for an Energy-Efficient Economy. 1: p. 1.171-1.183.

2. *Title 10, Code of Federal Regulations, Part 430- Energy Conservation Program for Consumer Products, Appendix E to Subpart B- Uniform Test Procedure for Measuring the Energy Consumption of Water Heaters,* January 1, 1998.